

MATERI MATEMATIKA PEMINATAN KELAS XI

PERTEMUAN KE-2

Petunjuk:

- 1) Pelajari contoh soal dan pembahasan berikut.
- 2) Kerjakan soal-soal tersebut di buku matematika peminatan
- 3) Hasil pekerjaan dikumpulkan di sekolah awal bulan Agustus



• Petunjuk Jawaban Soal Dasar •

1. Jika $\sin x = \sin \alpha$, nilai $x = \alpha + k \cdot 360^\circ$ atau $x = (180^\circ - \alpha) + k \cdot 360^\circ$.
2. Jika $\cos x = \cos \alpha$, nilai $x = \alpha + k \cdot 360^\circ$ atau $x = -\alpha + k \cdot 360^\circ$.
3. Jika $\tan x = \tan \alpha$, nilai $x = \alpha + k \cdot 180^\circ$.

Dengan k adalah bilangan bulat.

Atau,

1. Jika $\sin x = \sin \alpha$, nilai $x = \alpha + k \cdot 2\pi$ atau $x = (\pi - \alpha) + k \cdot 2\pi$.
2. Jika $\cos x = \cos \alpha$, nilai $x = \alpha + k \cdot 2\pi$ atau $x = -\alpha + k \cdot 2\pi$.
3. Jika $\tan x = \tan \alpha$, nilai $x = \alpha + k \cdot \pi$.

Dengan k adalah bilangan bulat.



• Contoh Soal dan Alternatif Penyelesaian •

1. Tentukan himpunan penyelesaian dari setiap persamaan trigonometri berikut!
 - a. $\sin x = \sin \frac{3}{8}\pi$, $0^\circ \leq x \leq 2\pi$
 - b. $\cos x = \cos 20^\circ$, $0^\circ \leq x \leq 360^\circ$
 - c. $\tan x = \tan \frac{5}{12}\pi$, $0^\circ \leq x \leq 2\pi$

Alternatif Penyelesaian

- a. $\sin x = \sin \frac{3}{8}\pi$, maka diperoleh:

$$1) \quad x = \frac{3}{8}\pi + k \cdot 2\pi$$

$$k = 0 \Rightarrow x = \frac{3}{8}\pi + 0 \cdot 2\pi = \frac{3}{8}\pi$$

$$k = 1 \Rightarrow x = \frac{3}{8}\pi + 1 \cdot 2\pi = \frac{19}{8}\pi \text{ (tidak memenuhi)}$$

$$2) \quad x = (\pi - \frac{3}{8}\pi) + k \cdot 2\pi \Leftrightarrow x = \frac{5}{8}\pi + k \cdot 2\pi$$

$$k = 0 \Rightarrow x = \frac{5}{8}\pi + 0 \cdot 2\pi = \frac{5}{8}\pi$$

$$k = 1 \Rightarrow x = \frac{5}{8}\pi + 1 \cdot 2\pi = \frac{21}{8}\pi \text{ (tidak memenuhi)}$$

Jadi, himpunan penyelesaiannya adalah $\{\frac{3}{8}\pi, \frac{5}{8}\pi\}$.

- b. $\cos x = \cos 20^\circ$, maka diperoleh:

$$1) \quad x = 20^\circ + k \cdot 360^\circ$$

$$k = 0 \Rightarrow x = 20^\circ + 0 \cdot 360^\circ = 20^\circ$$

$$k = 1 \Rightarrow x = 20^\circ + 1 \cdot 360^\circ = 380^\circ \text{ (tidak memenuhi)}$$

$$2) \quad x = -20^\circ + k \cdot 360^\circ$$

$$k = 0 \Rightarrow x = -20^\circ + 0 \cdot 360^\circ = -20^\circ \text{ (tidak memenuhi)}$$

$$k = 1 \Rightarrow x = -20^\circ + 1 \cdot 360^\circ = 340^\circ$$

$$k = 2 \Rightarrow x = -20^\circ + 2 \cdot 360^\circ = 700^\circ \text{ (tidak memenuhi)}$$

Jadi, himpunan penyelesaiannya adalah $\{20^\circ, 340^\circ\}$.

- c. $\tan x = \tan \frac{5}{12}\pi$, maka diperoleh:

$$x = \frac{5}{12}\pi + k \cdot \pi$$

$$k = 0 \Rightarrow x = \frac{5}{12}\pi + 0 \cdot \pi = \frac{5}{12}\pi$$

$$k = 1 \Rightarrow x = \frac{5}{12}\pi + 1 \cdot \pi = \frac{17}{12}\pi$$

$$k = 2 \Rightarrow x = \frac{5}{12}\pi + 2 \cdot \pi = \frac{29}{12}\pi \text{ (tidak memenuhi)}$$

Jadi, himpunan penyelesaiannya adalah $\{\frac{5}{12}\pi, \frac{17}{12}\pi\}$.

2. Tentukan himpunan penyelesaian dari persamaan-persamaan trigonometri berikut!

a. $\sin 5x = \sin 75^\circ, 0^\circ \leq x \leq 360^\circ$

b. $\cos 4x = \cos \frac{2}{5}\pi, 0 \leq x \leq 2\pi$

c. $\tan 3x = \tan 72^\circ, 0^\circ \leq x \leq 360^\circ$

Alternatif Penyelesaian

- a. $\sin 5x = \sin 75^\circ$, maka diperoleh:

$$1) \quad 5x = 75^\circ + k \cdot 360^\circ \Leftrightarrow x = 15^\circ + k \cdot 72^\circ$$

$$k = 0 \Rightarrow x = 15^\circ + 0 \cdot 72^\circ = 15^\circ$$

$$k = 1 \Rightarrow x = 15^\circ + 1 \cdot 72^\circ = 87^\circ$$

$$k = 2 \Rightarrow x = 15^\circ + 2 \cdot 72^\circ = 159^\circ$$

$$k = 3 \Rightarrow x = 15^\circ + 3 \cdot 72^\circ = 231^\circ$$

$$k = 4 \Rightarrow x = 15^\circ + 4 \cdot 72^\circ = 303^\circ$$

$$k = 5 \Rightarrow x = 15^\circ + 5 \cdot 72^\circ = 375^\circ \text{ (tidak memenuhi)}$$

$$2) \quad 5x = (180^\circ - 75^\circ) + k \cdot 360^\circ \Leftrightarrow 5x = 105^\circ + k \cdot 360^\circ \Rightarrow x = 21^\circ + k \cdot 72^\circ$$

$$k = 0 \Rightarrow x = 21^\circ + 0 \cdot 72^\circ = 21^\circ$$

$$k = 1 \Rightarrow x = 21^\circ + 1 \cdot 72^\circ = 93^\circ$$

$$k = 2 \Rightarrow x = 21^\circ + 2 \cdot 72^\circ = 165^\circ$$

$$k = 3 \Rightarrow x = 21^\circ + 3 \cdot 72^\circ = 237^\circ$$

$$k = 4 \Rightarrow x = 21^\circ + 4 \cdot 72^\circ = 309^\circ$$

$$k = 5 \Rightarrow x = 21^\circ + 5 \cdot 72^\circ = 381^\circ \text{ (tidak memenuhi)}$$

Jadi, himpunan penyelesaiannya adalah $\{15^\circ, 21^\circ, 87^\circ, 93^\circ, 159^\circ, 165^\circ, 231^\circ, 237^\circ, 303^\circ, 309^\circ\}$.

b. $\cos 4x = \cos \frac{2}{5}\pi$, maka diperoleh:

$$1) \quad 4x = \frac{2}{5}\pi + k \cdot 2\pi$$

$$x = \frac{1}{10}\pi + k \cdot \frac{1}{2}\pi$$

$$k = 0 \Rightarrow x = \frac{1}{10}\pi + 0 \cdot \frac{1}{2}\pi = \frac{1}{10}\pi$$

$$k = 1 \Rightarrow x = \frac{1}{10}\pi + 1 \cdot \frac{1}{2}\pi = \frac{3}{5}\pi$$

$$k = 2 \Rightarrow x = \frac{1}{10}\pi + 2 \cdot \frac{1}{2}\pi = \frac{11}{10}\pi$$

$$k = 3 \Rightarrow x = \frac{1}{10}\pi + 3 \cdot \frac{1}{2}\pi = \frac{8}{5}\pi$$

$$k = 4 \Rightarrow x = \frac{1}{10}\pi + 4 \cdot \frac{1}{2}\pi = \frac{21}{10}\pi \text{ (tidak memenuhi)}$$

$$2) \quad 4x = -\frac{2}{5}\pi + k \cdot 2\pi$$

$$x = -\frac{1}{10}\pi + k \cdot \frac{1}{2}\pi$$

$$k = 0 \Rightarrow x = -\frac{1}{10}\pi + 0 \cdot \frac{1}{2}\pi = -\frac{1}{10}\pi \text{ (tidak memenuhi)}$$

$$k = 1 \Rightarrow x = -\frac{1}{10}\pi + 1 \cdot \frac{1}{2}\pi = \frac{2}{5}\pi$$

$$k = 2 \Rightarrow x = -\frac{1}{10}\pi + 2 \cdot \frac{1}{2}\pi = \frac{9}{10}\pi$$

$$k = 3 \Rightarrow x = -\frac{1}{10}\pi + 3 \cdot \frac{1}{2}\pi = \frac{7}{5}\pi$$

$$k = 4 \Rightarrow x = -\frac{1}{10}\pi + 4 \cdot \frac{1}{2}\pi = \frac{19}{10}\pi$$

$$k = 5 \Rightarrow x = -\frac{1}{10}\pi + 5 \cdot \frac{1}{2}\pi = \frac{12}{5}\pi \text{ (tidak memenuhi)}$$

Jadi, himpunan penyelesaiannya adalah $\{\frac{1}{10}\pi, \frac{2}{5}\pi, \frac{3}{5}\pi, \frac{9}{10}\pi, \frac{11}{10}\pi, \frac{7}{5}\pi, \frac{8}{5}\pi, \frac{19}{10}\pi\}$.

c. $\tan 3x = \tan 72^\circ$, maka diperoleh:

$$3x = 72^\circ + k \cdot 180^\circ$$

$$x = 24^\circ + k \cdot 60^\circ$$

$$k = 0 \Rightarrow x = 24^\circ + 0 \cdot 60^\circ = 24^\circ$$

$$k = 1 \Rightarrow x = 24^\circ + 1 \cdot 60^\circ = 84^\circ$$

$$k = 2 \Rightarrow x = 24^\circ + 2 \cdot 60^\circ = 144^\circ$$

$$k = 3 \Rightarrow x = 24^\circ + 3 \cdot 60^\circ = 204^\circ$$

$$k = 4 \Rightarrow x = 24^\circ + 4 \cdot 60^\circ = 264^\circ$$

$$k = 5 \Rightarrow x = 24^\circ + 5 \cdot 60^\circ = 324^\circ$$

$$k = 6 \Rightarrow x = 24^\circ + 6 \cdot 60^\circ = 384^\circ \text{ (tidak memenuhi)}$$

Jadi, himpunan penyelesaiannya adalah $\{24^\circ, 84^\circ, 144^\circ, 204^\circ, 264^\circ, 324^\circ\}$.

3. Tentukan himpunan penyelesaian dari persamaan-persamaan trigonometri berikut!

a. $\sin(3x + \frac{1}{2}\pi) = \sin \frac{1}{6}\pi, 0 \leq x \leq 2\pi$

b. $\cos(x - 30^\circ) = \cos 25^\circ, 0^\circ \leq x \leq 360^\circ$

c. $\tan(2x - \frac{1}{4}\pi) = \tan \frac{1}{3}\pi, 0 \leq x \leq 2\pi$

Alternatif Penyelesaian

a. $\sin(3x + \frac{1}{2}\pi) = \sin \frac{1}{6}\pi$, maka diperoleh:

$$1) \quad 3x + \frac{1}{2}\pi = \frac{1}{6}\pi + k \cdot 2\pi$$

$$3x = -\frac{1}{3}\pi + k \cdot 2\pi \Leftrightarrow x = -\frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$$

$$k = 0 \Rightarrow x = -\frac{1}{9}\pi + 0 \cdot \frac{2}{3}\pi = \frac{1}{9}\pi \text{ (tidak memenuhi)}$$

$$k = 1 \Rightarrow x = -\frac{1}{9}\pi + 1 \cdot \frac{2}{3}\pi = \frac{5}{9}\pi$$

$$k = 2 \Rightarrow x = -\frac{1}{9}\pi + 2 \cdot \frac{2}{3}\pi = \frac{11}{9}\pi$$

$$k = 3 \Rightarrow x = -\frac{1}{9}\pi + 3 \cdot \frac{2}{3}\pi = \frac{17}{9}\pi$$

$$k = 4 \Rightarrow x = -\frac{1}{9}\pi + 4 \cdot \frac{2}{3}\pi = \frac{23}{9}\pi \text{ (tidak memenuhi)}$$

2) $3x + \frac{1}{2}\pi = (\pi - \frac{1}{6}\pi) + k \cdot 2\pi$

$$3x + \frac{1}{2}\pi = \frac{5}{6}\pi + k \cdot 2\pi$$

$$3x = \frac{1}{3}\pi + k \cdot 2\pi$$

$$x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$$

$$k = 0 \Rightarrow x = \frac{1}{9}\pi + 0 \cdot \frac{2}{3}\pi = \frac{1}{9}\pi$$

$$k = 1 \Rightarrow x = \frac{1}{9}\pi + 1 \cdot \frac{2}{3}\pi = \frac{7}{9}\pi$$

$$k = 2 \Rightarrow x = \frac{1}{9}\pi + 2 \cdot \frac{2}{3}\pi = \frac{13}{9}\pi$$

$$k = 3 \Rightarrow x = \frac{1}{9}\pi + 3 \cdot \frac{2}{3}\pi = \frac{19}{9}\pi \text{ (tidak memenuhi)}$$

Jadi, himpunan penyelesaiannya adalah $\{\frac{1}{9}\pi, \frac{5}{9}\pi, \frac{7}{9}\pi, \frac{11}{9}\pi, \frac{13}{9}\pi, \frac{17}{9}\pi\}$.

b. $\cos(x - 30^\circ) = \cos 25^\circ$, maka diperoleh:

1) $x - 30^\circ = 25^\circ + k \cdot 360^\circ$

$$x = 55^\circ + k \cdot 360^\circ$$

$$k = 0 \Rightarrow x = 55^\circ + 0 \cdot 360^\circ = 55^\circ$$

$$k = 1 \Rightarrow x = 55^\circ + 1 \cdot 360^\circ = 415^\circ \text{ (tidak memenuhi)}$$

2) $x - 30^\circ = -25^\circ + k \cdot 360^\circ$

$$x = 5^\circ + k \cdot 360^\circ$$

$$k = 0 \Rightarrow x = 5^\circ + 0 \cdot 360^\circ = 5^\circ$$

$$k = 1 \Rightarrow x = 5^\circ + 1 \cdot 360^\circ = 365^\circ \text{ (tidak memenuhi)}$$

Jadi, himpunan penyelesaiannya adalah $\{5^\circ, 55^\circ\}$.

c. $\tan(2x - \frac{1}{4}\pi) = \tan \frac{1}{3}\pi$, maka diperoleh:

$$2x - \frac{1}{4}\pi = \frac{1}{3}\pi + k \cdot \pi$$

$$2x = \frac{7}{12}\pi + k \cdot \pi \Leftrightarrow x = \frac{7}{24}\pi + k \cdot \frac{1}{2}\pi$$

$$k = 0 \Rightarrow x = \frac{7}{24}\pi + 0 \cdot \frac{1}{2}\pi = \frac{7}{24}\pi$$

$$k = 1 \Rightarrow x = \frac{7}{24}\pi + 1 \cdot \frac{1}{2}\pi = \frac{19}{24}\pi$$

$$k = 2 \Rightarrow x = \frac{7}{24}\pi + 2 \cdot \frac{1}{2}\pi = \frac{31}{24}\pi$$

$$k = 3 \Rightarrow x = \frac{7}{24}\pi + 3 \cdot \frac{1}{2}\pi = \frac{43}{24}\pi$$

$$k = 4 \Rightarrow x = \frac{7}{24}\pi + 4 \cdot \frac{1}{2}\pi = \frac{55}{24}\pi \text{ (tidak memenuhi)}$$

Jadi, himpunan penyelesaiannya adalah $\{\frac{7}{24}\pi, \frac{19}{24}\pi, \frac{31}{24}\pi, \frac{43}{24}\pi\}$.

4. Tentukan himpunan penyelesaian dari persamaan-persamaan trigonometri berikut!

a. $\cos 2x = \sin x, 0^\circ \leq x \leq 360^\circ$

b. $\sin 2x = \cos x, 0 \leq x \leq 2\pi$

Alternatif Penyelesaian

a. $\cos 2x = \sin x = \cos(90^\circ - x)$, maka diperoleh:

1) $2x = (90^\circ - x) + k \cdot 360^\circ$

$$3x = 90^\circ + k \cdot 360^\circ \Leftrightarrow x = 30^\circ + k \cdot 120^\circ$$

$$k = 0 \Rightarrow x = 30^\circ + 0 \cdot 120^\circ = 30^\circ$$

$$k = 1 \Rightarrow x = 30^\circ + 1 \cdot 120^\circ = 150^\circ$$

$$k = 2 \Rightarrow x = 30^\circ + 2 \cdot 120^\circ = 270^\circ$$

$$k = 3 \Rightarrow x = 30^\circ + 3 \cdot 120^\circ = 390^\circ \text{ (tidak memenuhi)}$$

2) $2x = -(90^\circ - x) + k \cdot 360^\circ$

$$x = -90^\circ + k \cdot 360^\circ$$

$$k = 0 \Rightarrow x = -90^\circ + 0 \cdot 360^\circ = -90^\circ \text{ (tidak memenuhi)}$$

$$k = 1 \Rightarrow x = -90^\circ + 1 \cdot 360^\circ = 270^\circ$$

$$k = 2 \Rightarrow x = -90^\circ + 2 \cdot 360^\circ = 630^\circ \text{ (tidak memenuhi)}$$

Jadi, himpunan penyelesaiannya adalah $\{30^\circ, 150^\circ, 270^\circ\}$.

b. $\sin 2x = \cos x = \sin(\frac{1}{2}\pi - x)$, maka diperoleh:

1) $2x = (\frac{1}{2}\pi - x) + k \cdot 2\pi$

$$3x = \frac{1}{2}\pi + k \cdot 2\pi \Leftrightarrow x = \frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$$

$$k = 0 \Rightarrow x = \frac{1}{6}\pi + 0 \cdot \frac{2}{3}\pi = \frac{1}{6}\pi$$

$$k = 1 \Rightarrow x = \frac{1}{6}\pi + 1 \cdot \frac{2}{3}\pi = \frac{5}{6}\pi$$

$$k = 2 \Rightarrow x = \frac{1}{6}\pi + 2 \cdot \frac{2}{3}\pi = \frac{3}{2}\pi$$

$$k = 3 \Rightarrow x = \frac{1}{6}\pi + 3 \cdot \frac{2}{3}\pi = \frac{13}{6}\pi \text{ (tidak memenuhi)}$$

2) $2x = (\pi - (\frac{1}{2}\pi - x)) + k \cdot 2\pi$

$$2x = (\frac{1}{2}\pi + x) + k \cdot 2\pi \Leftrightarrow x = \frac{1}{2}\pi + k \cdot 2\pi$$

$$k = 0 \Rightarrow x = \frac{1}{2}\pi + 0 \cdot 2\pi = \frac{1}{2}\pi$$

$$k = 1 \Rightarrow x = \frac{1}{2}\pi + 1 \cdot 2\pi = \frac{5}{2}\pi \text{ (tidak memenuhi)}$$

Jadi, himpunan penyelesaiannya adalah $\{\frac{1}{6}\pi, \frac{1}{2}\pi, \frac{5}{6}\pi, \frac{3}{2}\pi\}$.

2. Menggunakan tabel nomor 1, tentukan himpunan penyelesaian dari setiap persamaan trigonometri berikut!

a. $\sin x = \sin 30^\circ$

Jawab: _____

b. $\cos x = \cos 45^\circ$

Jawab: _____

c. $\tan x = \tan 60^\circ$

Jawab: _____

d. $\sin x = \sin 150^\circ$

Jawab: _____

e. $\cos x = \cos 240^\circ$

Jawab: _____

f. $\tan x = \tan 315^\circ$

Jawab: _____

SELAMAT MENGERJAKAN...